



Description Logic

Summer Semester 2022

Exercise Sheet 7

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Prof. Dr.-Ing. Franz Baader, Dr. Oliver Fernández Gil

Exercise 7.1 Recall the tableau algorithm for $\mathcal{ALCN}$ with blocking, and the example (in slide 48) that shows non-termination when using the $\leq$ -rule *without pruning*. At each step of the algorithm, write down the multiset $\mu(\mathcal{A})$ for the current ABox $\mathcal{A}$, and explain why the termination proof fails in this case.

Exercise 7.2 Prove that the tableau algorithm for $\mathcal{ALCN}$ is sound and complete.

Exercise 7.3 We extend the tableau algorithm from $\mathcal{ALCN}$ to $\mathcal{ALCQ}$ by modifying the $\geq$ -rule and the $\leq$ -rule as follows:

The $\geq$ -rule

Condition: $\mathcal{A}$ contains $a:(\geq n r.C)$, but there are no n distinct individuals $b_1, \dots, b_n$ with $\{(a, b_i):r, b_i:C \mid 1 \leq i \leq n\} \subseteq \mathcal{A}$, and a is not blocked

Action: $\mathcal{A} \longrightarrow \mathcal{A} \cup \{(a, d_i):r, d_i:C \mid 1 \leq i \leq n\} \cup \{d_i \neq d_j \mid 1 \leq i < j \leq n\}$, where $d_1, \dots, d_n$ are new individual names

The $\leq$ -rule

Condition: $\mathcal{A}$ contains $a:(\leq n r.C)$, and there are $n+1$ distinct individuals $b_0, \dots, b_n$ with $\{(a, b_i):r, b_i:C \mid 0 \leq i \leq n\} \subseteq \mathcal{A}$

Action: $\mathcal{A} \longrightarrow \text{prune}(\mathcal{A}, b_j)[b_j \mapsto b_i] \cup \{b_i = b_j\}$ for $i \neq j$ such that, if b_j is a root individual, then so is b_i

For the knowledge base

$$(\{C \sqsubseteq E\}, \{a:(\leq 1 r.(D \sqcap E)), (a, b):r, b:C \sqcap D, (a, c):r, c:D \sqcap E, c:\neg C\}),$$

determine whether it is consistent, and whether the proposed algorithm detects this.